Geodesy 1B
Lecture 4
Height Systems

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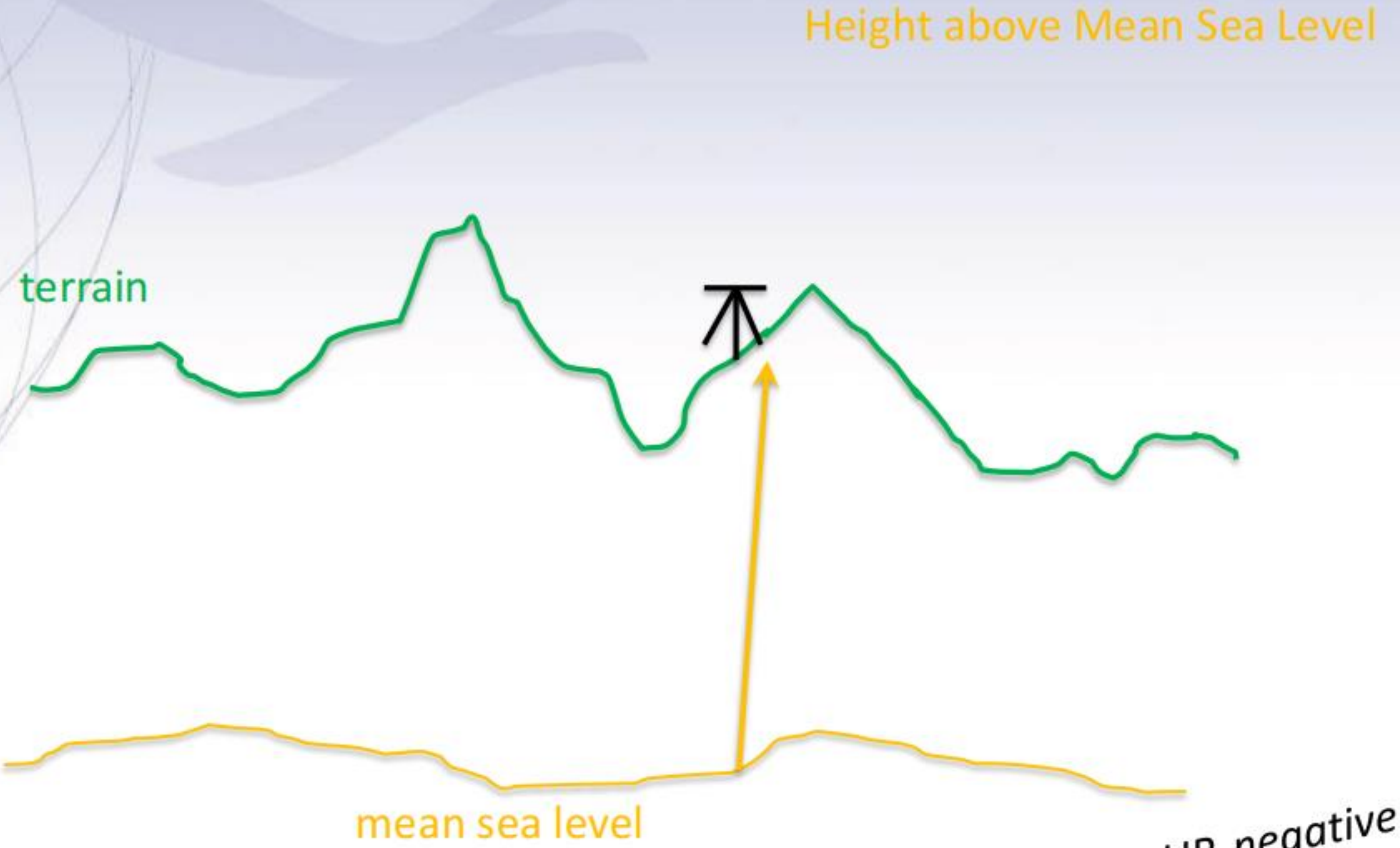
- Height systems
- Vertical datums used in US vs Egypt
- Important websites and sources

HEIGHTS

Geodesy

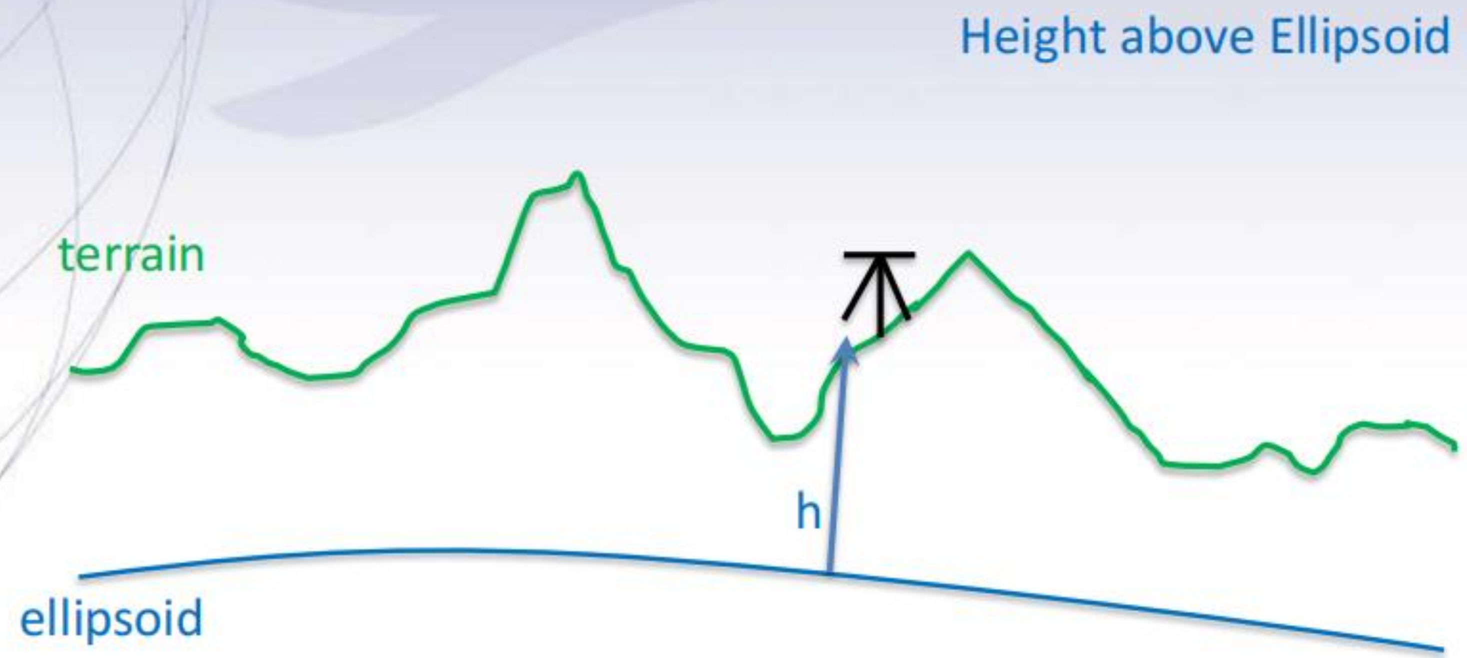
- Heights above or below a reference surface
  - Mean Sea Level Heights
  - Ellipsoid Heights
  - Orthometric Heights
  - Geoid Heights
  - Vertical Datums
    - 1912
    - NGVD29
    - NAVD88
  - Dynamic Heights

Geodesy



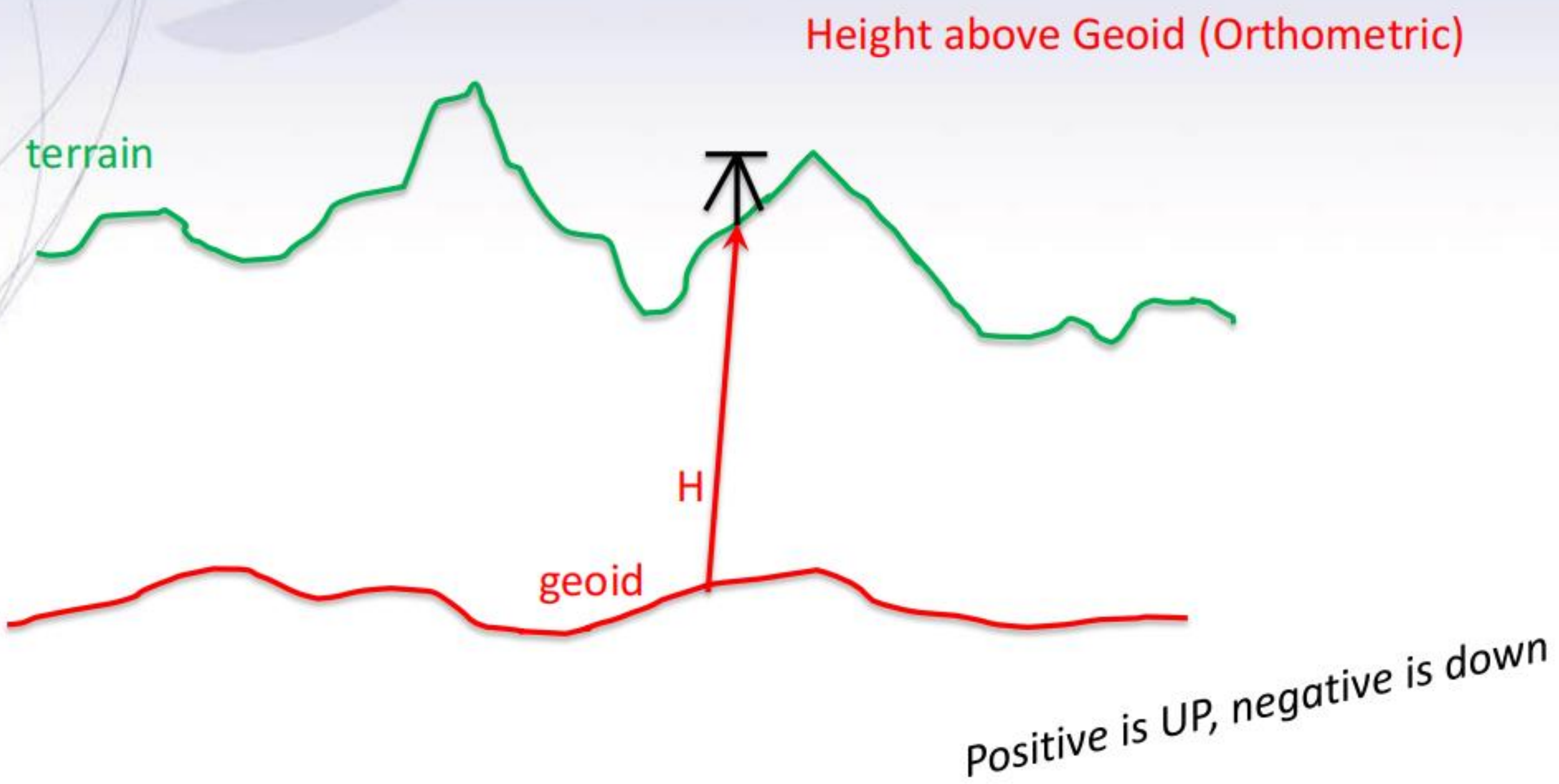
Positive is UP, negative is down

Geodesy



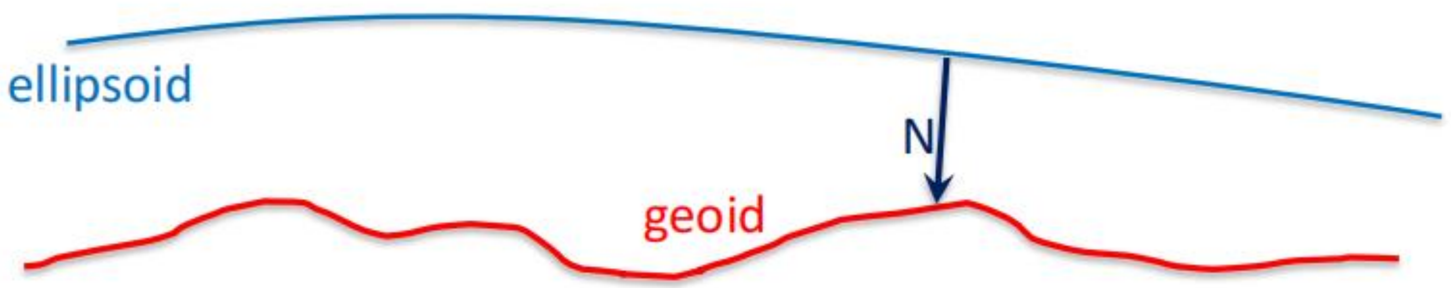
Positive is UP, negative is down

Geodesy


Height above Geoid (Orthometric)
terrain
H
geoid
Positive is UP, negative is down

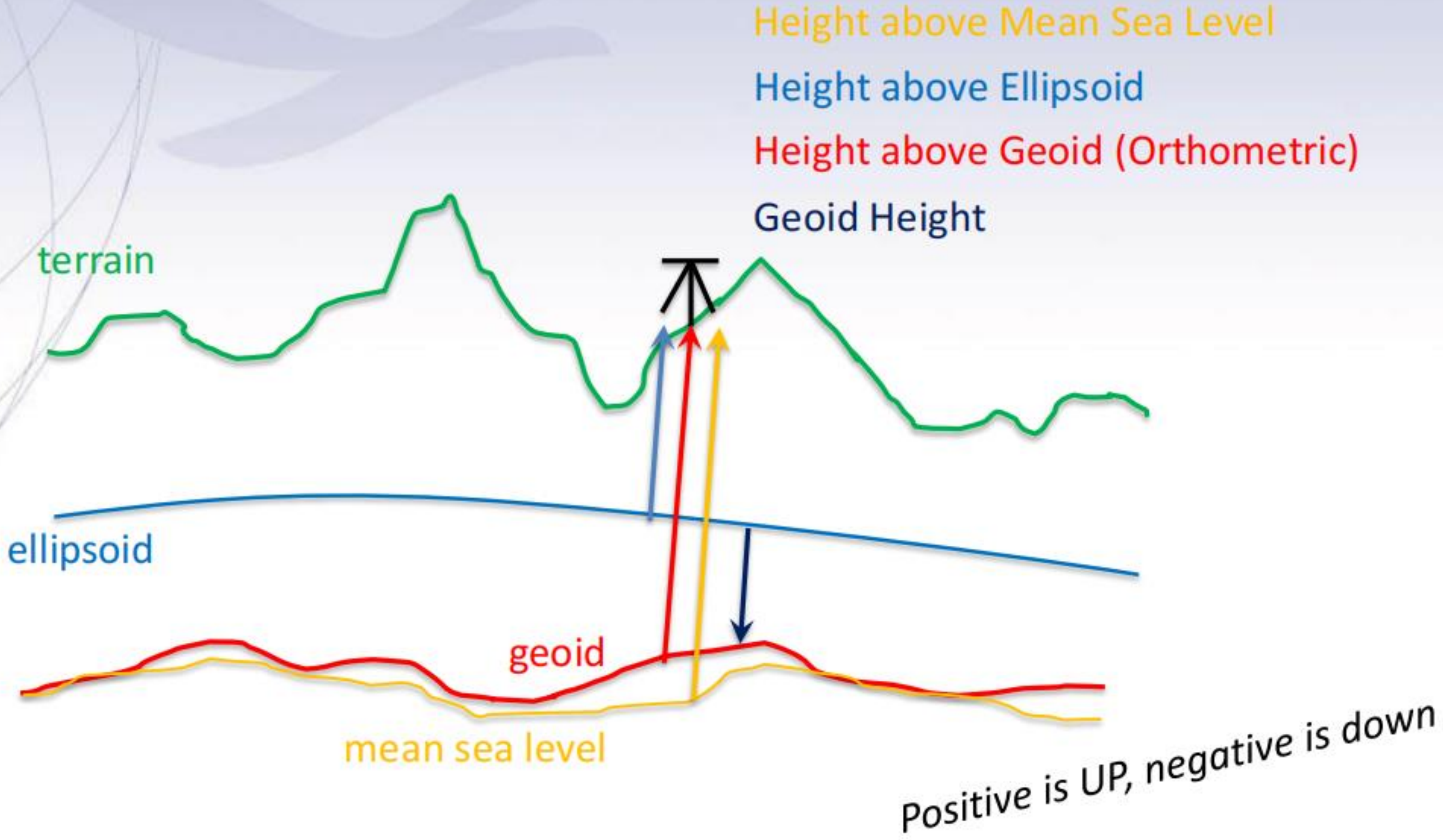
Geodesy

Geoid Height above ellipsoid

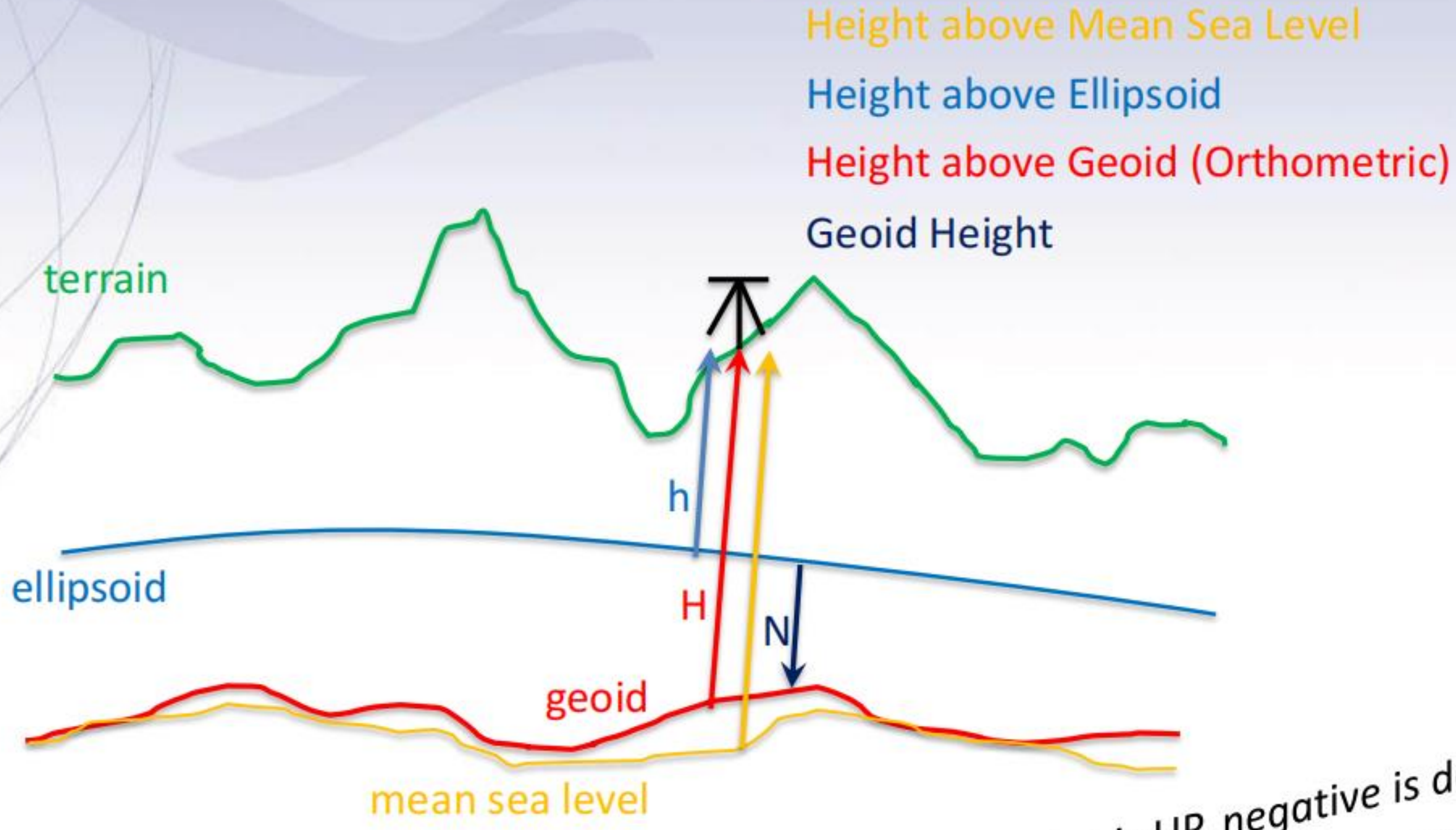


Positive is UP, negative is down

Geodesy


Height above Mean Sea Level
Height above Ellipsoid
Height above Geoid (Orthometric)
Geoid Height
terrain
ellipsoid
geoid
mean sea level
Positive is UP, negative is down

Geodesy



$$H = h - N$$

Positive is UP, negative is down

Note that N is negative in most of CONUS.
Note that h may be negative near coastlines.
Could H be negative?

Geodesy

- Vertical Datums in the United States
  - There have been several vertical datums in use across the United States. The earliest ones were “local”, since the leveling did not form an interconnected network.
  - The earliest unified datums were the 1912 Datum (4th General Adjustment).
  - Followed by the NGVD29 adjustment.
  - Followed by the NAVD88 adjustment.

What about Vertical Datums in Egypt?

Task2: Report about the vertical datums in Egypt and challenges and problems for local Egyptian geoid

Height Systems

Height Systems

- Orthometric
- Normal (orthometric normal)
- Dynamic
- Ellipsoidal

Variety of height systems (datums) used requires careful definition of differences and transformation among the systems

Which height is most used in surveying?

- The height reference that is mostly used in surveying job is orthometric
- Orthometric height is also commonly provided on topographic maps
- Thus, even though ellipsoidal heights are much simpler to determine (eg. GPS) we still need to determine orthometric heights

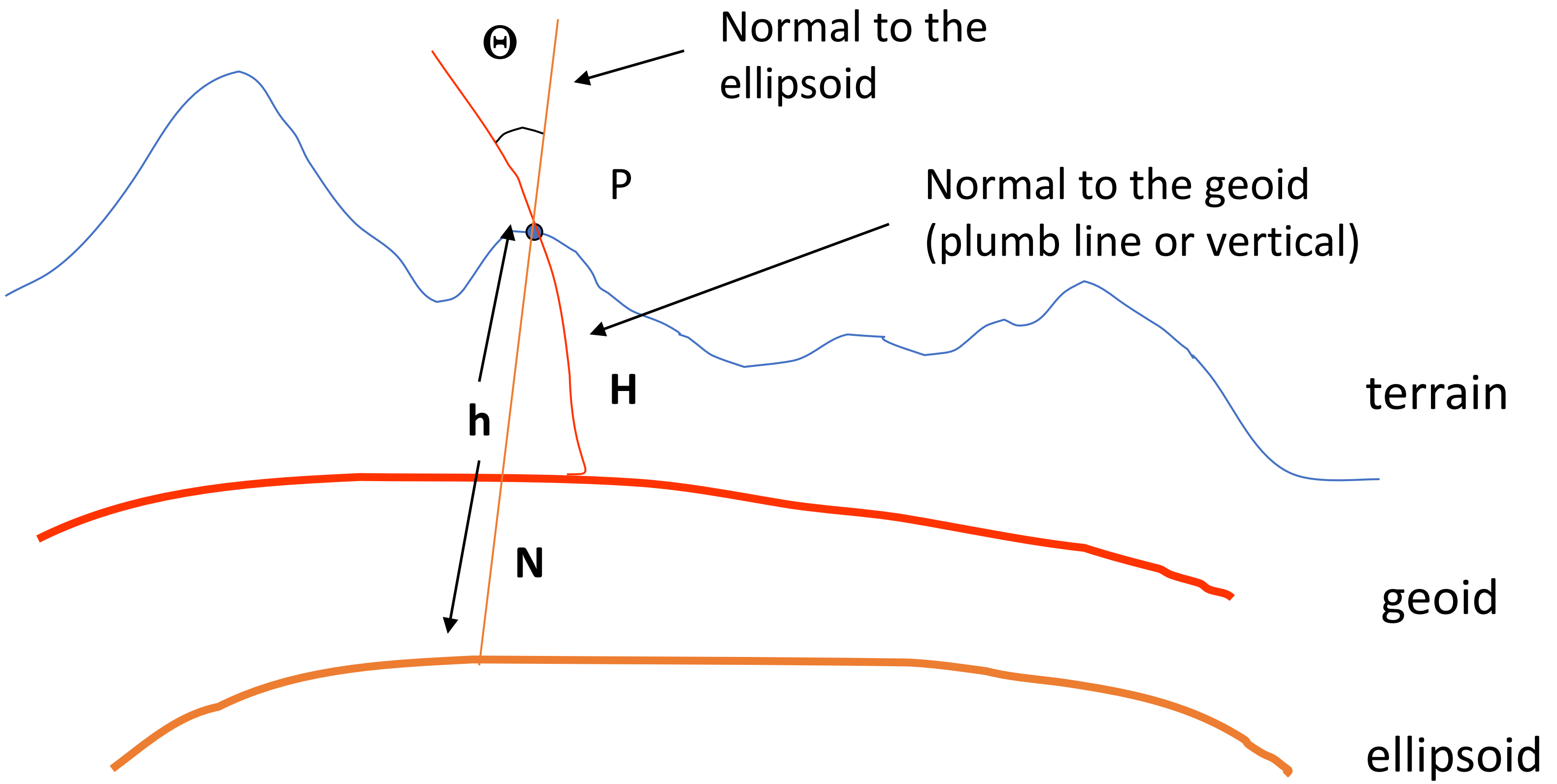
Θ - angle between the normal to the ellipsoid and the vertical direction (normal

to the geoid), so-called deflection of the vertical

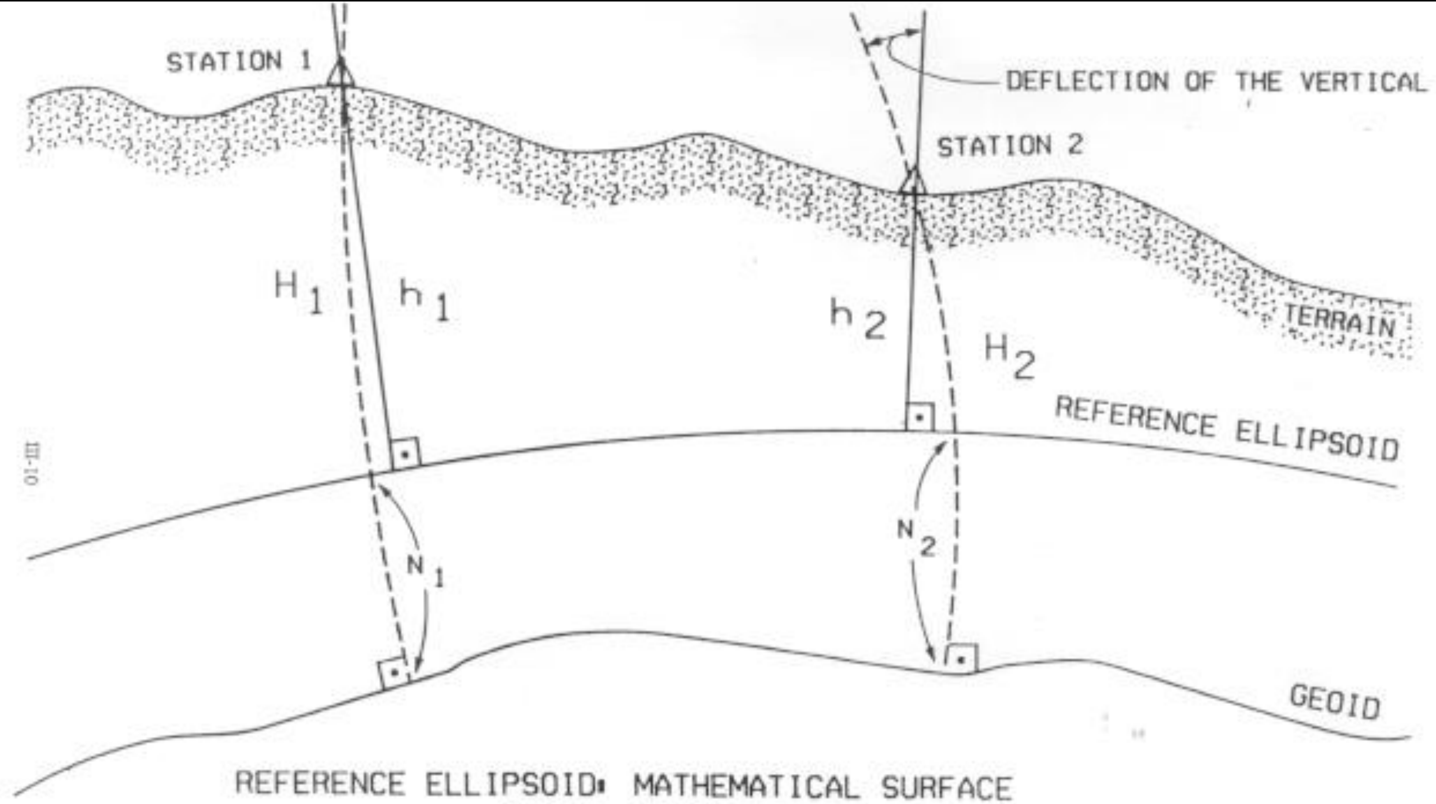
H – orthometric height

h – ellipsoidal height **h = H + N**

N – geoid undulation (computed from geoid model provided by NGS)



Orthometric vs Ellipsoidal Height



REFERENCE ELLIPSOID: MATHEMATICAL SURFACE

GEOID: NOMINALLY EQUATED TO DISTORTED SEA-LEVEL

TO DETERMINE ELEVATIONS USING GPS:

$h = H + N$

H = ELEVATIONS (MEASURED FROM GEOID) (Orthometric height)

h = ELLIPSOIDAL HEIGHT (FROM GPS)

N = GEOID SEPARATION FROM ELLIPSOID (computed from a geoid model)

So, how do we determine orthometric height?

- By spirit leveling
- And gravity observations along the leveling path, or
- Recently -- GPS combined with geoid models (easy!!!) but not as accurate as spirit leveling + gravity observations

$$H = h - N$$

But why do we need gravity observations with spirit leveling?

Because the sum of the measured height differences along the leveling path between points A and B is not equal to the difference in orthometric height between points A and B

Why?

Level Surfaces and Plumb Lines 1/2

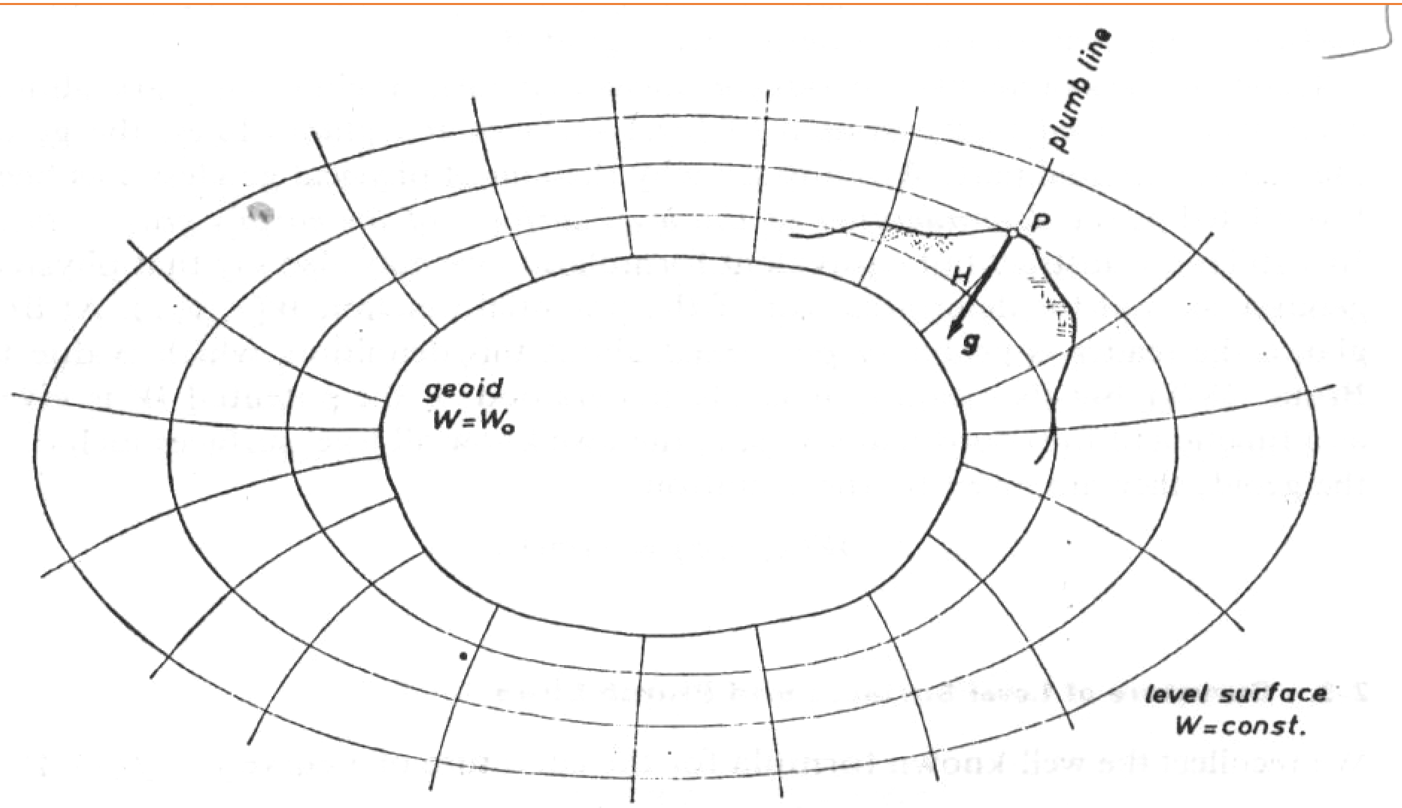


FIGURE 2-2

Level surfaces and plumb lines.

Equipotential surfaces are not parallel to each other

Level Surfaces and Plumb Lines 2/2

- The level surfaces are, so to speak, **horizontal everywhere**, they share the geodetic importance of the plumb line, because they are normal to it
- Plumb lines (line of forces, vertical lines) are curved
- **Orthometric heights are measured along the curved plumb lines**
- Equipotential surfaces are rather complicated mathematically and they are **not parallel to each other**
- Consequently:
  - **Orthometric heights are not constant on the equipotential surface !**
  - **Thus, points on the same level surface would have different orthometric height !**

Spirit leveling

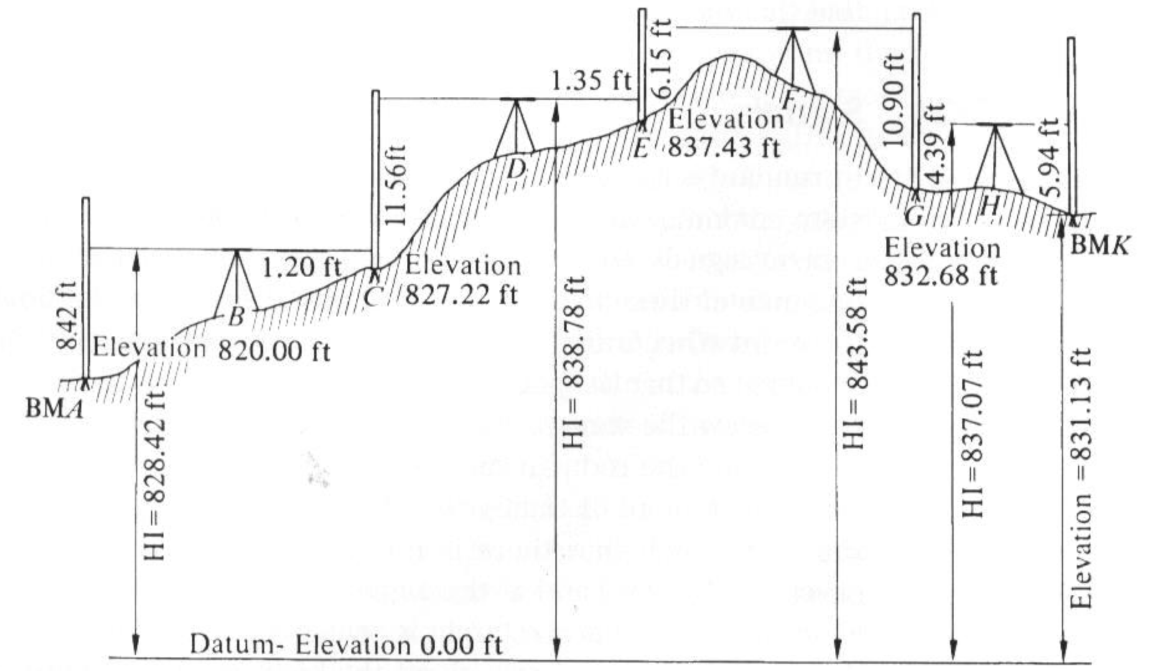


FIGURE 3-36 Direct leveling.

Height differences between the consecutive locations of backward and forward rods correspond to the local separation between the level surfaces through the bottom of the rods, measured along the plumb line direction

Orthometric Height vs. Spirit Leveling

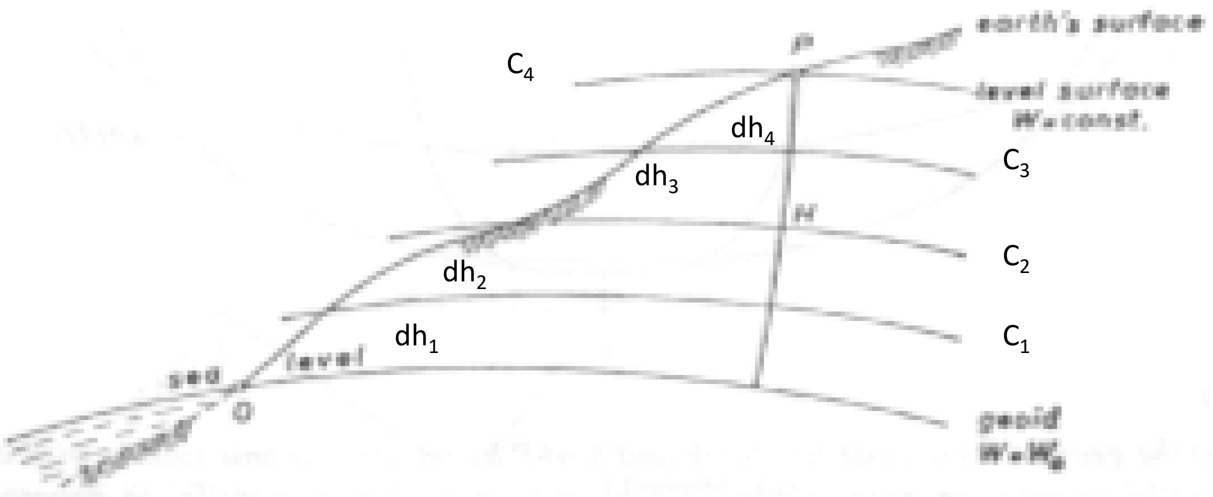


FIGURE 2-8

The orthometric height H.

$\Sigma dh_i \neq H$

C_1, C_2, C_3, C_4 – geopotential numbers corresponding to level (equipotential) surfaces

dh_1, dh_2, dh_3, dh_4 – height difference between the level surfaces (determined by spirit leveling, path-dependent); their sum is not equal to H !

Because equipotential surfaces are **not parallel to each other**

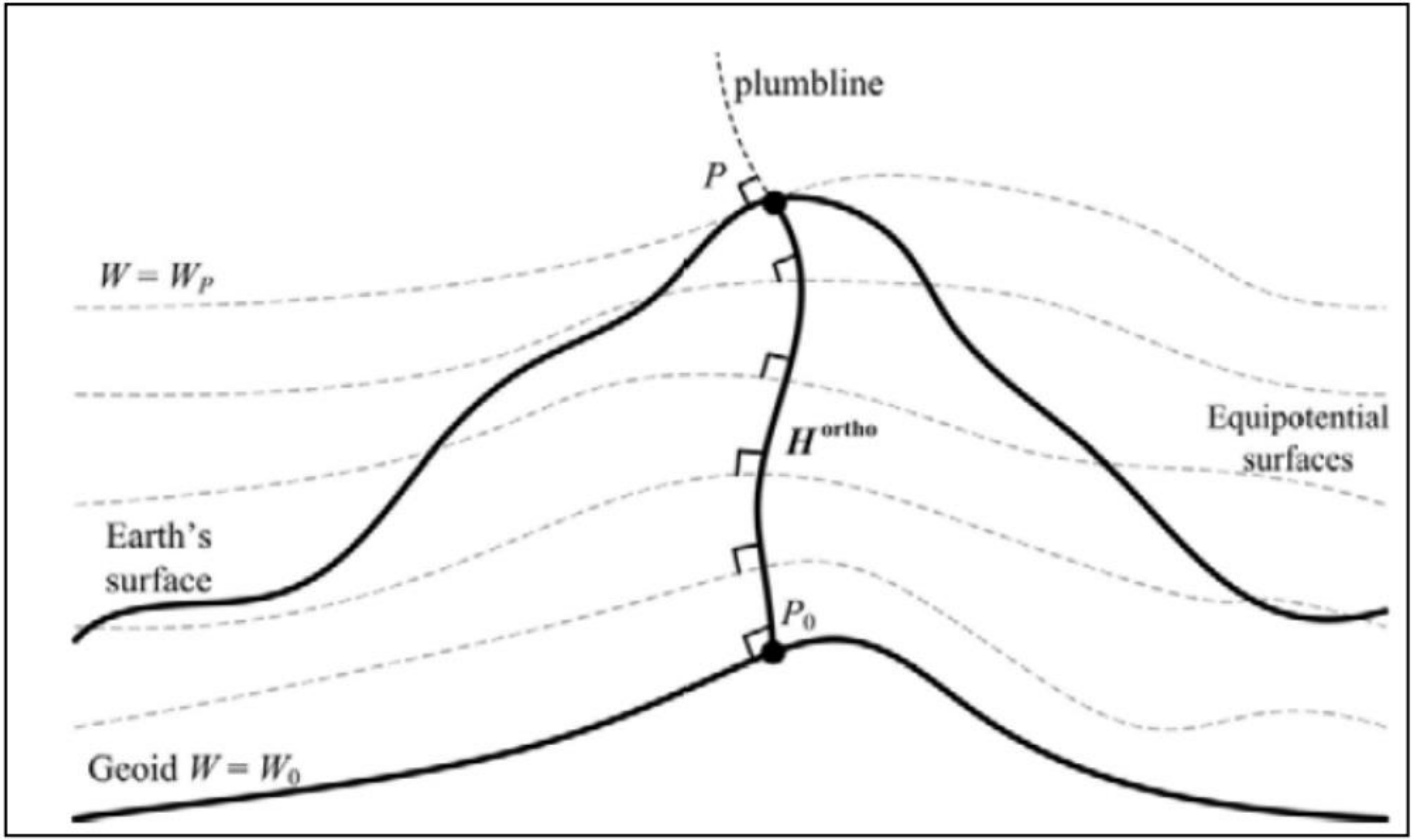


Figure (1.5): The orthometric height (H^{ortho}) of P [2].

Geopotential Numbers 1/3

- The difference in height, dh, measured during each set up of leveling can be **converted to a difference in potential** by multiplying dh by the mean value of gravity, g_m, for the set up (along dh).

geopotential difference = g_m*dh

- **Geopotential number C**, or potential difference between the geoid level W_0 and the geopotential surface W_P through point P on the Earth surface (see Figure 2-8), is defined as

$$\int_0^P g dh = C = W_0 - W_P$$

Where g is the gravity value along the leveling path. This formula is used to compute C when g is measured, and is independent on the path of integration!

Geopotential Numbers 2/3

Since the **computation of C is not path-dependent**, the geopotential number can be also expressed as

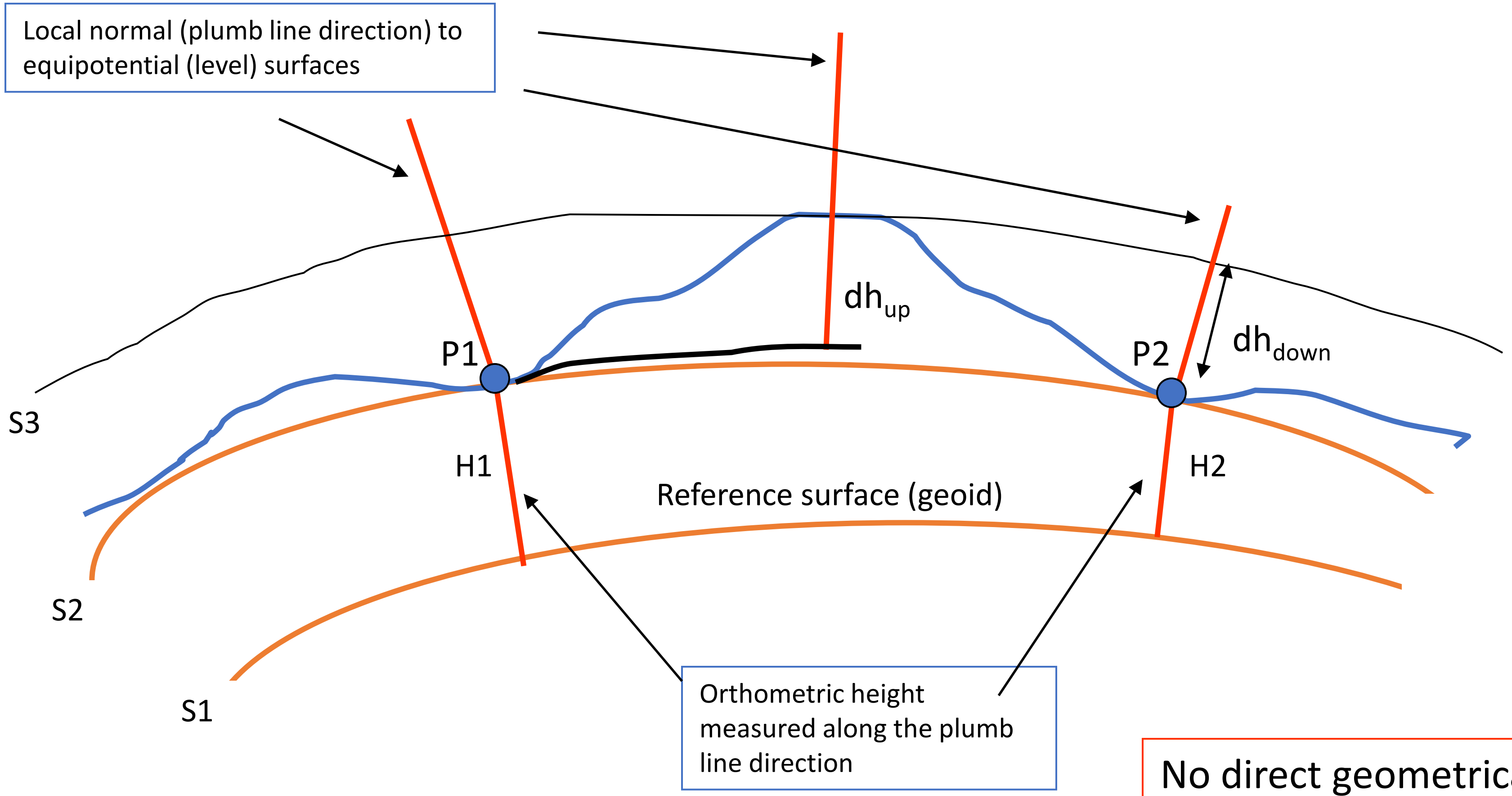
$$C = g_m * H,$$

where H is the height above the geoid (mean sea level) and g_m represents the mean value of gravity along H (along the plumb line at point P on Figure 2-8; see "orthometric height vs. spirit leveling)

- the last relationship justifies the units for C being kgal*meter; it is not used to determine C!
- Finally:
  - **Geopotential number is constant for the geopotential (level) surface**
  - **Consequently, geopotential numbers can be used to define height and are considered a natural measure for height**

REMEMBER: Orthometric heights are not constant on the equipotential surface !

- Observed difference in height depends on leveling route
- Points on the same level surface have different orthometric heights



$\Delta H = H1 - H2 \neq dh_{up} + dh_{down} \neq 0$

No direct geometrical relation between the results of leveling and orthometric heights

What then, if not orthometric height, is directly obtained by leveling?

- If gravity is also measured, then geopotential numbers, C (defined by the integral formula shown earlier), result from leveling
- Thus, leveling combined with gravity measurements furnishes potential difference, that is, physical quantities
- **Consequently, orthometric height are considered as quantities derived from potential differences**
- Thus, leveling without gravity measurements introduces error (for short lines might be neglected) to orthometric height

Geopotential Numbers 3/3

Let's summarize:

- The sum of leveled height differences between two pints, A and B, on the Earth surface will not equal to the difference in the orthometric heights H_A and H_B
- The difference in height, dh, measured during each set up of leveling depends on the route taken, as level (equipotential) surfaces are not parallel to each other
- Consequently, based on the leveling and gravity measurements
    - the geopotential numbers are initially estimated (using the integral formula introduced earlier), based on the leveling and gravity measurements along the leveling path
    - geopotential numbers can then be converted to heights (orthometric, normal or dynamic – see definitions below) if gravity value along the plumb line through surface point P is known

Height = C/gravity

Height Systems 1/5

- **In order to convert the results of leveling to orthometric heights we need gravity inside the earth (along the plumb line)**
  - since we cannot measure it directly, as the reference surface lies within the Earth, beneath the point, we use special formulas to compute the mean value of gravity, along the plumb line, based on the surface gravity measured at point P
  - reduction formulas used to compute the mean gravity, g_m, based on gravity measured at point P on the Earth surface lead to:
    - Orthometric height, (**$H = C/g_m$**) or
- **The reduction formula used to compute mean gravity, based on normal gravity at point P on the Earth surface leads to**:
    - Normal (also called normal orthometric) height, (**$H^* = C/\gamma_m$**)

Where γ is so-called normal gravity (model) corresponding to the gravity field of an ellipsoid of reference (Earth best fitting ellipsoid), and subscript "m" stands for "mean"

Height Systems 2/5

- We can also define dynamic heights
  - use normal gravity, γ_{45}, defined on the ellipsoid at 45 degree latitude, ($H_D = C/\gamma_{45}$)

- **Note:** term "normal gravity" always refers to the gravity defined for the reference ellipsoid, while "gravity" relates to geoid or Earth itself

Height Systems 3/5

Sometimes, instead of formulas provided above (involving C), it is convenient to use correction terms and apply them to the sum of leveled height differences:

- Consequently, the measured elevation difference has to be corrected using so-called orthometric correction to obtain orthometric height (height above the geoid)

  Max orthometric correction is about 15 cm per 1 km of measured height difference

- Or, the measured elevation difference has to be corrected using so-called dynamic correction to obtain dynamic height (no geometric meaning and factual reference surface; defined mathematically)

- Or, normal correction is used to derive normal heights

- All corrections need gravity information along the leveling path (equivalent to computation of C based on gravity observations!)

Height Systems 4/5

- Dynamic heights are constant for the level surface, and have no geometric meaning
- Orthometric height
  - differs for points on the same level surface because the level surfaces are not parallel. This gives rise to the well-known paradoxes of “water flowing uphill”
  - measured along the curved plumb line with respect to geoid level
- Normal height of point P on earth surface is a geometric height above the reference ellipsoid of the point Q on the plumb line of P such as normal gravity potential and Q is the same as actual gravity potential at P.
  - measured along the normal plumb line (“normal” refers to the line of force direction in the gravity field of the reference ellipsoid (model))
- All above types of heights are derived from geopotential numbers

Height Systems 5/5

- A disadvantage of orthometric and normal heights is that neither indicates the direction of flow of water. Only dynamic heights possess this property.
- That is, two points with identical dynamic heights are on the same equipotential surface of the actual gravity field, and water will not flow from one to the other point.
- Two points with identical orthometric heights lie on different equipotential surfaces and water will flow from one point to the other, even though they have the same orthometric height
- The last statement holds for normal heights, although due to the smoothness of the normal gravity field, the effect is not as severe

Vertical Datums

Vertical Datum Definition 1/2

- **Horizontal control networks** provide positional information (latitude and longitude) with reference to a mathematical surface called sphere or spheroid (ellipsoid)
- By contrast, **vertical control networks** provide elevation with reference to a surface of constant gravitational potential, called geoid (approximately mean see level)
    - this type of elevation information is called **orthometric height (height above the geoid or mean sea level**) determined by spirit leveling (including gravity measurements and reduction formulas).
- Height information referenced to the ellipsoidal surface is called **ellipsoidal height**. This kind of height information is provided by GPS
- Points on or near the Earth's surface commonly are associated with three coordinates, a **latitude**, a **longitude**, and a **height**.

Vertical Datum Definition 2/2

- **Vertical datum** is defined by the surface of reference – geoid or ellipsoid
- An access to the vertical datum is provided by a **vertical control network** (similar to the network of reference points furnishing the access to the horizontal datums)
- Vertical control network is defined as an interconnected system of bench marks
- Why do we need vertical control network?
  - to reduce amount of leveling required for surveying job
  - to provide backup for destroyed bench marks
  - to assist in monitoring local changes
  - to provide a common framework

Vertical Datums: NGVD 29 and NAVD 88

- **NGVD 29 – National Geodetic Vertical Datum of 1929**
  - defined by heights of 26 tidal stations in US and Canada
  - uses normal orthometric height (based on normal gravity formula)
- **NAVD 88 – North American Vertical Datum of 1988**
  - defined by one height (Father Point/Rimouski, Quebec, Canada)
  - 585,000 permanent bench marks
  - uses Helmert orthometric height (based on Helmert gravity formula)
  - removed systematic errors and blunders present in the earlier datum
  - orthometric height compatible with GPS-derived height using geoid model
  - improved set of heights on single vertical datum for North America

Vertical Datums: NGVD 29 and NAVD 88

- **Difference between NGVD 29 and NAVD 88**
  - ranges between – 40 cm to 150 cm
  - in Alaska between 94 and 240 cm
  - in most stable areas the difference stays around 1 cm
  - accuracy of datum conversion is 1-2 cm, may exceed 2.5 cm
  - transformation procedures and software provided by NGS (www.ngs.noaa.gov)

International Great Lake Datum (IGLD) 1985

- **IGLD 85**
  - replaced earlier IGLD 1955
  - defined by one height (Father Point/Rimouski, Quebec, Canada)
  - uses dynamic height (based on normal gravity at 45 degrees latitude)
  - virtually identical to NAVD 88 but published in dynamic heights!

Vertical Datums

- Use of proper vertical datum (reference surface) is very important
- Never mix vertical datums as ellipsoid – geoid separation can reach 100 m!
- Geoid undulation, N, is provided by models (high accuracy, few centimeters in the most recent model) developed by the National Geodetic Survey (NGS) and published on their web page

www.ngs.noaa.gov

So, in order to derive the height above the see level (H) with GPS observations – determine the ellipsoidal height (h) with GPS and apply the geoid undulation (N) according to the formula $H = h - N$

Important websites and sources

Online geoid height calculator

unavco.org/software/geodetic-utilities/geoid-height-calculator/geoid-height-calculator.html

GAGE | EarthScope Consortium

DATA INSTRUMENTATION SOFTWARE KNOWLEDGE BASE
WHAT WE DO EVENTS EDUCATION NEWS

home › software › geodetic utilities › geoid height calculator

Software

- Help with Software
- **Geodetic Utilities**
  - Plate Motion Calculator
  - **Geoid Height Calculator**

Geoid Height Calculator

Our Geoid Height Calculator calculates a geoid undulation at a point(s) whose latitude and longitude is specified. This new version (2021-08-18) of our program is designed to use a variety of Earth Gravitational Models and can now calculate heights for multiple points. This calculator is an online notebook developed by UNAVCO and included here.

We have included the entire notebook (all cells) with all the notes, references and explication, because this tool is often used in educational settings. For regular users please ignore the bottom half of the scrollable page.

Geoid Height Calculator

https://www.unavco.org/software/geodetic-utilities/geoid-height-calculator/geoid-height-calculator.html

جمهورية مصر العربية
وزارة الموارد المائية والرى
الهيئة المصرية العامة للمساحة

المواصفات الفنية لأعمال المساحة والخرائط

الجزء الأول

أعمال المساحة

الاصدار الأول
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